Two-Way ANOVA in SPSS

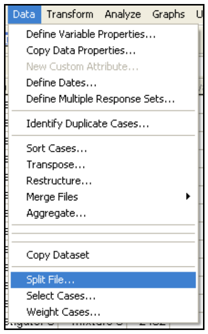
Preliminary research on the production of imitation pearls entailed studying the effect of the number of coats of a special lacquer applied to an opalescent plastic bead used as the base of the pearl on the market value of the pearl. Four batches of beads (12 beads per batch) were used in the study, and it is desired to also consider the batch effect on the market value. The three levels of lacquer coatings (6, 8, and 10 coats) were fixed in advance, as were the production formulations for the four batches. The market value of each pearl was determined by a panel of experts. The data are shown in the accompanying table. Perform the appropriate analysis of variance procedure (including a profile plot of the means, hypothesis tests, and multiple comparisons) for this experiment. Use $\alpha = 0.05$.

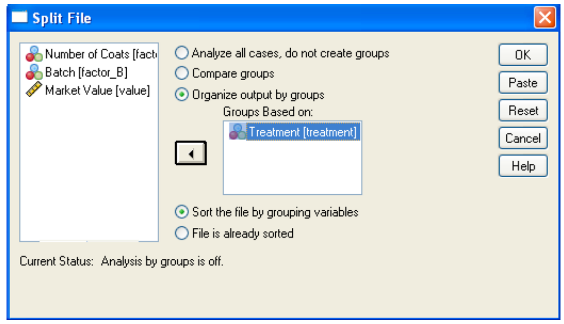
Number of Coats	*Batch*			
	1	*2*	*3*	*4*
6	72.0	72.1	75.2	70.4
	74.6	76.9	73.8	68.1
	67.4	74.8	75.7	72.4
	72.8	73.3	77.8	72.4
8	76.9	80.3	80.2	74.3
	78.1	79.3	76.6	77.6
	72.9	76.6	77.3	74.4
	74.2	77.2	79.9	72.9
10	76.3	80.9	79.2	71.6
	74.1	73.7	78.0	77.7
	77.1	78.6	77.6	75.2
	75.0	80.2	81.2	74.4

1. Each treatment is a combination of the Number of Coats value and the Batch value. Enter the treatment numbers into one column with the corresponding number of coats, batch number, and market values into separate variables *(see figure, below)*. Be sure to enter your variables appropriately.

	treatment	factor_A	factor_B	value
1	Treatment 1	6 Coats	Batch 1	72.0
2	Treatment 1	6 Coats	Batch 1	74.6
3	Treatment 1	6 Coats	Batch 1	67.4
4	Treatment 1	6 Coats	Batch 1	72.8
5	Treatment 2	6 Coats	Batch 2	72.1
6	Treatment 2	6 Coats	Batch 2	76.9
7	Treatment 2	6 Coats	Batch 2	74.8
8	Treatment 2	6 Coats	Batch 2	73.3
9	Treatment 3	6 Coats	Batch 3	75.2
10	Treatment 3	6 Coats	Batch 3	73.8
11	Treatment 3	6 Coats	Batch 3	75.7
12	Treatment 3	6 Coats	Batch 3	77.8
13	Treatment 4	6 Coats	Batch 4	70.4
14	Treatment 4	6 Coats	Batch 4	68.1
15	Treatment 4	6 Coats	Batch 4	72.4
16	Treatment 4	6 Coats	Batch 4	72.4
17	Treatment 5	8 Coats	Batch 1	76.9
18	Treatment 5	8 Coats	Batch 1	78.1
19	Treatment 5	8 Coats	Batch 1	72.9
20	Treatment 5	8 Coats	Batch 1	74.2
21	Treatment 6	8 Coats	Batch 2	80.3
22	Treatment 6	8 Coats	Batch 2	79.3
23	Treatment 6	8 Coats	Batch 2	76.6
24	Treatment 6	8 Coats	Batch 2	77.2
25	Treatment 7	8 Coats	Batch 3	80.2
26	Treatment 7	8 Coats	Batch 3	76.6
27	Treatment 7	8 Coats	Batch 3	77.3
28	Treatment 7	8 Coats	Batch 3	79.9
29	Treatment 8	8 Coats	Batch 4	74.3
30	Treatment 8	8 Coats	Batch 4	77.6
31	Treatment 8	8 Coats	Batch 4	74.4
32	Treatment 8	8 Coats	Batch 4	72.9
33	Treatment 9	10 Coats	Batch 1	76.3
34	Treatment 9	10 Coats	Batch 1	74.1
35	Treatment 9	10 Coats	Batch 1	77.1
36	Treatment 9	10 Coats	Batch 1	75.0
37	Treatment 10	10 Coats	Batch 2	80.9
38	Treatment 10	10 Coats	Batch 2	73.7
39	Treatment 10	10 Coats	Batch 2	78.6
40	Treatment 10	10 Coats	Batch 2	80.2
41	Treatment 11	10 Coats	Batch 3	79.2
42	Treatment 11	10 Coats	Batch 3	78.0
43	Treatment 11	10 Coats	Batch 3	77.6
44	Treatment 11	10 Coats	Batch 3	81.2
45	Treatment 12	10 Coats	Batch 4	71.6
46	Treatment 12	10 Coats	Batch 4	77.7
47	Treatment 12	10 Coats	Batch 4	75.2
48	Treatment 12	10 Coats	Batch 4	74.4

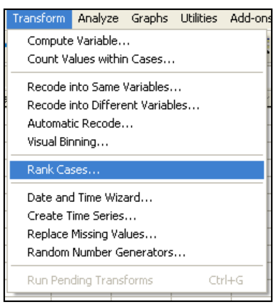
2. Now it is time to check the normality assumption [this won't work if you have m = 1 observation per treatment combination]. Select "Split File" from the "Data" menu so that we can tell SPSS that we want separate Q–Q Plots for each treatment group *(see left figure, below)*. Select "Organize output by groups" and enter "treatment" as the variable that groups are based upon *(see right figure, below)*. Now click "OK".

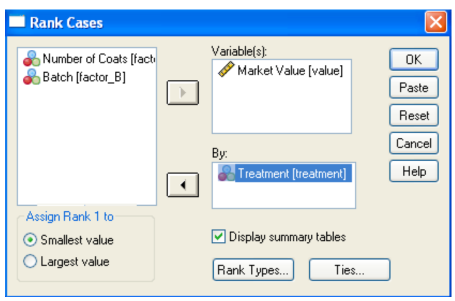


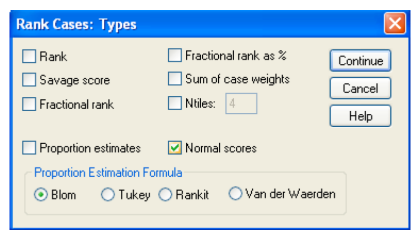


You can create separate Normal Q–Q Plots to assess the normality of each treatment group *(see separate handout on Normal Q–Q Plots)*, or you can create a single matrix of Normal Q–Q Plots using the following procedure (a single matrix plot takes up much less space than many separate Normal Q–Q Plots.

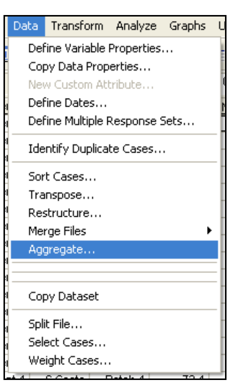
Select "Rank Cases…" from the "Transform" menu *(see left figure, below)*. Enter "Market Value" as the variable and rank the cases by "Treatment" *(see middle figure, below)*. Click the "Rank Types…" button and be sure that "Normal scores" and "Blom" are selected in the resulting window, then click the "Continue" button *(see right figure, below)*. Now click "OK".

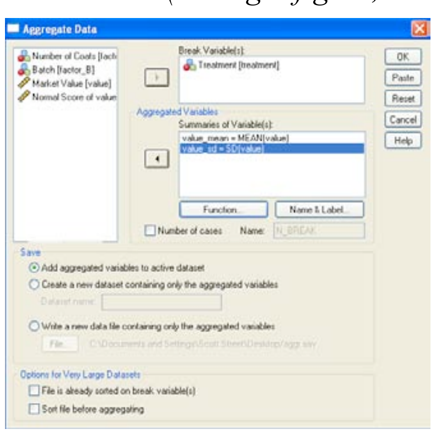


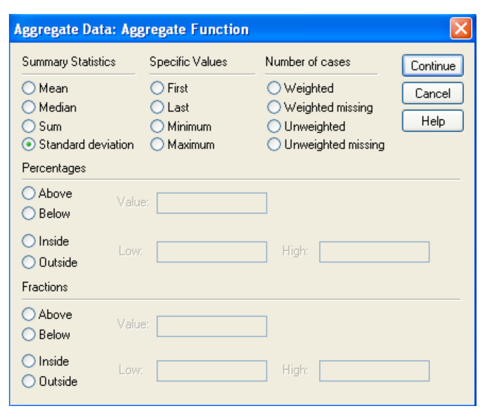




Select "Aggregate…" from the "Data" menu *(see left figure, below)*. Enter "Treatment" in the break variable box and "Market Value" in the summaries of variables box (MEAN) will be selected by default *(see middle figure, below)*. Enter "Market Value" in the summaries of variables box again, then click the "Function…" button and select "Standard Deviation" before clicking the "Continue" button *(see right figure, below)*. Now click "OK".

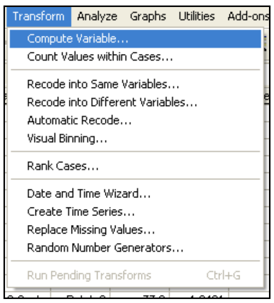


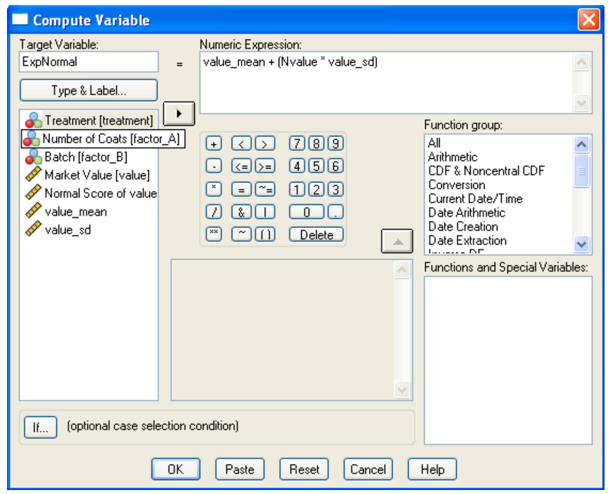




We have just computed the normal scores for the data values within each treatment group as well as the mean and standard deviation for each treatment group. We will now use these values to compute the expected observed values if the data were normally distributed (these values are what we plot versus the actual observed values in a Normal Q–Q Plot).

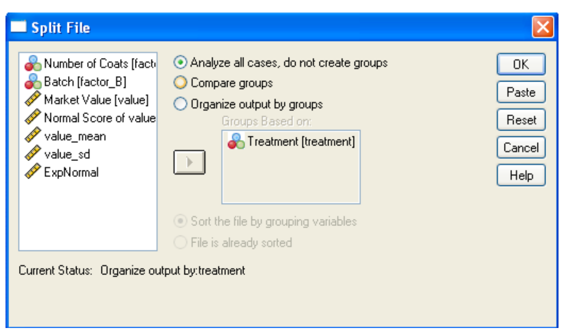
Select "Compute Variable…" from the "Transform" menu *(see left figure, below)*. Enter "ExpNormal" as the target variable, and the numeric expression should be "value_mean + (Nvalue ∗ value_sd)" *(see right figure, below)*.



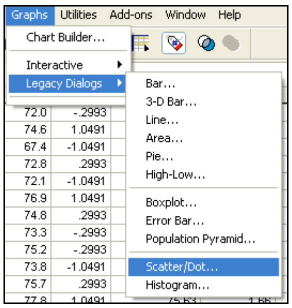


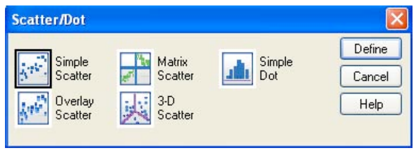
Since a normal score (*z*-score or "Nvalue") gives the expected number of standard deviations away from the mean that a data value should fall if it were normal, we compute the expected observed values by determining what value is the *z*-score number of standard deviations from the mean ($x = \mu + z\sigma$), where the mean and standard deviation are estimated for each treatment group using the corresponding sample mean and sample standard deviation, respectively. Now click "OK".

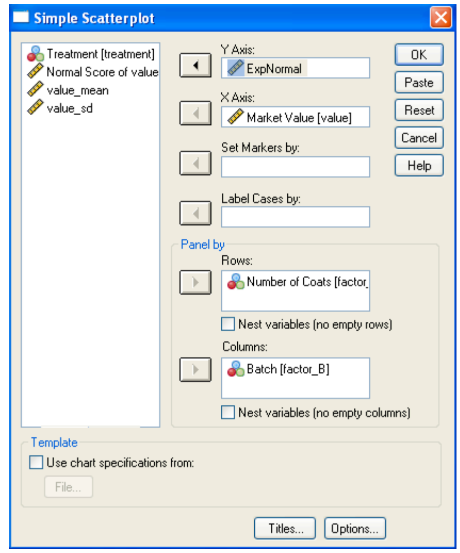
Once you've created the expected observed values ("ExpNormal"), select "Split File" from the "Data" menu and then select "Analyze all cases, do not create groups" in order to return SPSS to its normal data analysis mode *(see figure, below)*.



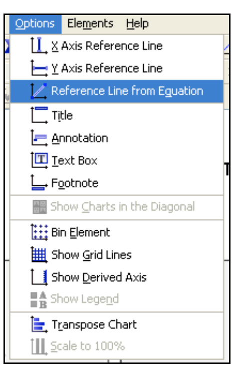
Select “Scatter/Dot…” from the “Legacy Dialogs” submenu of the “Graphs” menu *(see left figure, below)*. Select “Simple Scatter” then click the “Define” button *(see middle figure, below)*. Enter “ExpNormal” in the Y-Axis box, “Market Value” in the X-Axis box, and panel your plots by “Number of Coats” as the rows and “Batch” as the columns (to match the way the data are presented in the problem description) *(see right figure, below)*. Be sure to click the “Titles…” button and enter “Q-Q Plots by Treatment Groups” as Line 1 of your title.

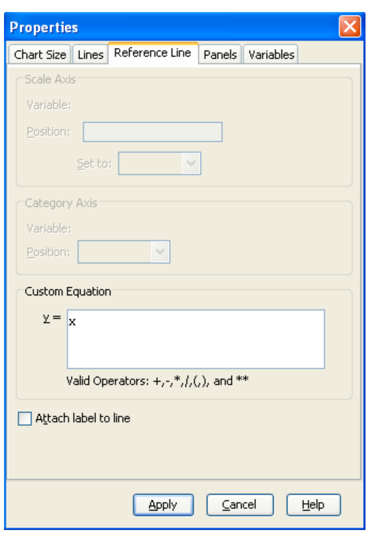


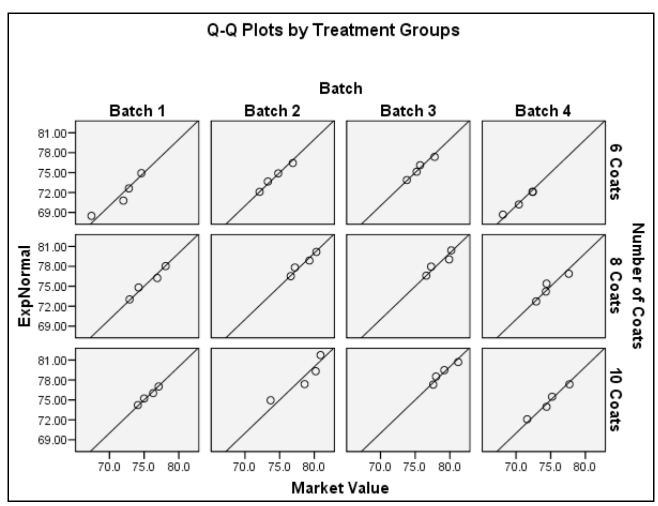




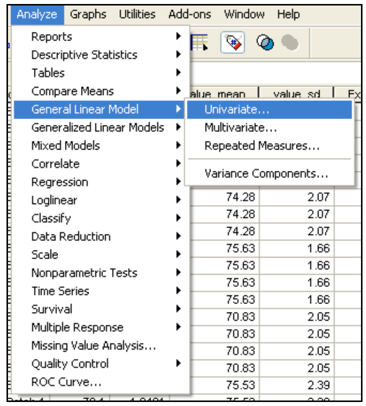
Once you click “OK”, you’ll be switched to the output window to see your plots so far (we aren’t done quite yet). Now, double-click the graph matrix to open it in editor mode. From the “Options” menu in editor mode, select “Reference Line from Equation” *(see left figure, below)*. Lines should be added to each plot, and the “Properties” window should open. Change the custom equation so that the equation plotted becomes “y = x” *(see middle figure, below)*. You may now exit out of editor mode as your plots now have the proper diagonal line displayed *(see right figure, below)*. The resulting plots can now be interpreted individually in the same manner as Q–Q plots created by the usual method (Analyze → Descriptive Statistics → Q-Q Plots…) are interpreted.



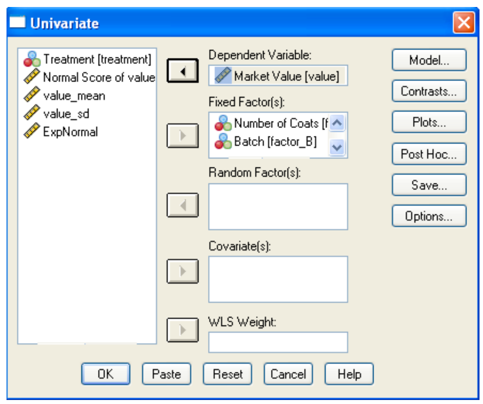




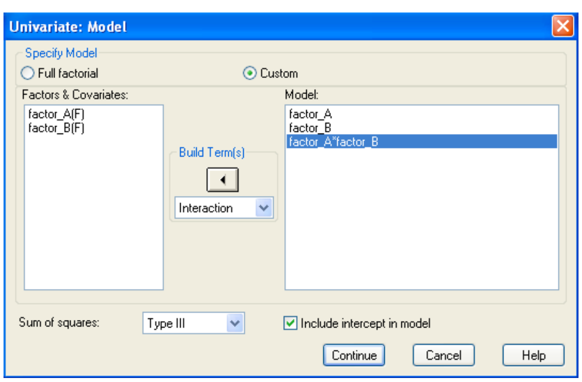
3. Select Analyze → General Linear Model → Univariate… *(see figure, below)*.

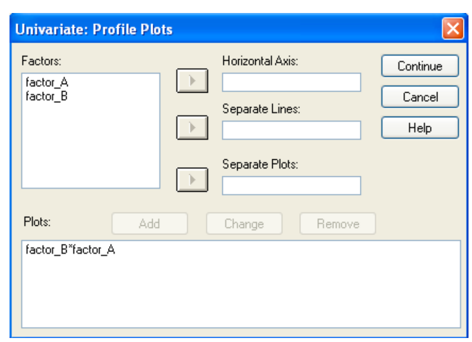


4. Select "Market Value" as the dependent variable, and select "Number of Coats" (factor_A) and "Batch Number" (factor_B) as the fixed factors *(see figure, below)*.



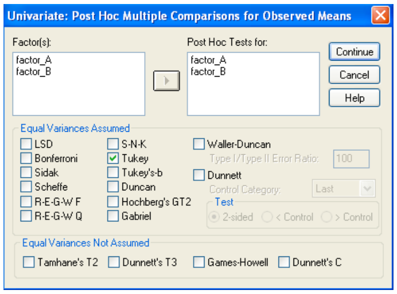
Click the "Model…" button. In the Univariate:Model window, select the "Custom" option and then the pull-down option in the center for "Main effects". Select "factor_A" (Number of Coats) and "factor_B" (Batch) and move them to be in the Model. Next, after changing the pull-down option in the center to "Interaction", select both "factor_A" and "factor_B" and then move them to be in the Model. [*Note:* You will not include this "Interaction" component if you have $m = 1$ observation per treatment combination.] Also, be sure that "Type III" sum of squares and "Include intercept in model" are selected, and then click "Continue" *(see left figure, below)*.

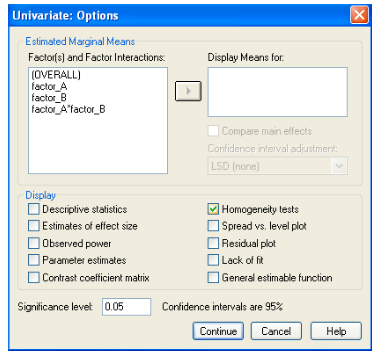




Click the "Plots…" button. In the Profile Plots window, select one factor for the horizontal axis and one for the separate lines (the treatment means will be on the vertical axis). Be sure to click "Add" and then click "Continue" *(see right figure, above)*.

Click the "Post Hoc…" button, select the "Tukey" procedure, enter "factor_A" (Number of Coats) and "factor_B" (Batch) as the Post Hoc Tests variables, and click "Continue". Click the "Options…" button, select "Homogeneity tests" [Levene's Test won't work unless $m \geq 3$], enter 0.05 for the significance level (95% CI corresponds to a 5% (0.05) significance level), and click "Continue" *(see 2 figures, below)*. Now click the "OK" button in the main Univariate window.





5. Your output should look like this.

Univariate Analysis of Variance

Between-Subjects Factors

		Value Label	N
Number of Coats	1	6 Coats	16
	2	8 Coats	16
	3	10 Coats	16
Batch	1	Batch 1	12
	2	Batch 2	12
	3	Batch 3	12
	4	Batch 4	12

Levene's Test of Equality of Error Variances[a]

Dependent Variable: Market Value

F	df1	df2	Sig.
.4994	11	36	.8909

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

a. Design: Intercept+factor_A+factor_B+factor_A * factor_B

Tests of Between-Subjects Effects

Dependent Variable: Market Value

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	305.0917[a]	11	27.7356	5.7508	.0000
Intercept	274397.7633	1	274397.7633	56894.57	.0000
factor_A	150.3879	2	75.1940	15.5910	.0000
factor_B	152.8517	3	50.9506	10.5643	.0000
factor_A * factor_B	1.8521	6	.3087	.0640	.9988
Error	173.6250	36	4.8229		
Total	274876.4800	48			
Corrected Total	478.7167	47			

a. R Squared = .637 (Adjusted R Squared = .526)

Post Hoc Tests

Number of Coats

Multiple Comparisons

Dependent Variable: Market Value
Tukey HSD

(I) Number of Coats	(J) Number of Coats	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
6 Coats	8 Coats	-3.6875*	.7764	.0001	-5.5854	-1.7896
	10 Coats	-3.8188*	.7764	.0001	-5.7166	-1.9209
8 Coats	6 Coats	3.6875*	.7764	.0001	1.7896	5.5854
	10 Coats	-.1313	.7764	.9844	-2.0291	1.7666
10 Coats	6 Coats	3.8188*	.7764	.0001	1.9209	5.7166
	8 Coats	.1313	.7764	.9844	-1.7666	2.0291

Based on observed means.

*. The mean difference is significant at the 0.05 level.

Homogeneous Subsets

Market Value

Tukey HSD[a,b]

Number of Coats	N	Subset 1	Subset 2
6 Coats	16	73.1063	
8 Coats	16		76.7938
10 Coats	16		76.9250
Sig.		1.0000	.9844

Means for groups in homogeneous subsets are displayed.
Based on Type III Sum of Squares
The error term is Mean Square(Error) = 4.823.

a. Uses Harmonic Mean Sample Size = 16.000.

b. Alpha = 0.05.

Batch

Multiple Comparisons

Dependent Variable: Market Value
Tukey HSD

(I) Batch	(J) Batch	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Batch 1	Batch 2	-2.7083*	.8966	.0229	-5.1230	-.2937
	Batch 3	-3.4250*	.8966	.0027	-5.8396	-1.0104
	Batch 4	.8333	.8966	.7893	-1.5813	3.2480
Batch 2	Batch 1	2.7083*	.8966	.0229	.2937	5.1230
	Batch 3	-.7167	.8966	.8542	-3.1313	1.6980
	Batch 4	3.5417*	.8966	.0019	1.1270	5.9563
Batch 3	Batch 1	3.4250*	.8966	.0027	1.0104	5.8396
	Batch 2	.7167	.8966	.8542	-1.6980	3.1313
	Batch 4	4.2583*	.8966	.0002	1.8437	6.6730
Batch 4	Batch 1	-.8333	.8966	.7893	-3.2480	1.5813
	Batch 2	-3.5417*	.8966	.0019	-5.9563	-1.1270
	Batch 3	-4.2583*	.8966	.0002	-6.6730	-1.8437

Based on observed means.

*. The mean difference is significant at the 0.05 level.

Homogeneous Subsets

Market Value

Tukey HSD[a,b]

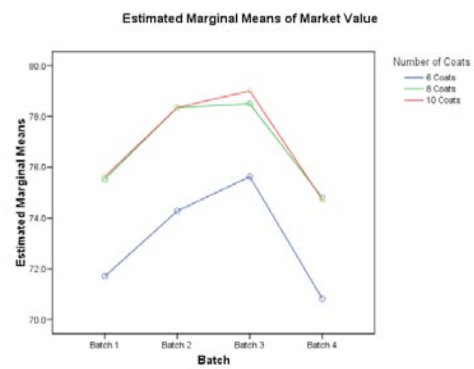
Batch	N	Subset 1	Subset 2
Batch 4	12	73.4500	
Batch 1	12	74.2833	
Batch 2	12		76.9917
Batch 3	12		77.7083
Sig.		.7893	.8642

Means for groups in homogeneous subsets are displayed.
Based on Type III Sum of Squares
The error term is Mean Square(Error) = 4.823.

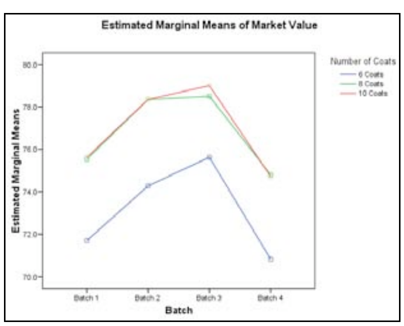
a. Uses Harmonic Mean Sample Size = 12.000.

b. Alpha = 0.05.

Profile Plots



6. Since the lines of the profile plot (below) each follow the same basic pattern with no substantial crossings, we conclude that there are probably no significant interaction effects.



7. Levene's Test for Homogeneity of Variances and Normal Q-Q Plots.

Step 0: **Check Assumptions of Equal Variances (Homogeneity of Variances) and Normality**
The Levene Statistic p-value = 0.8909 is greater than $\alpha = 0.05$ (from Step 2), so we fail to reject the null hypothesis that the variances are all equal. Since the points in each plot appear to closely follow the respective diagonal lines, then the populations are approximately normally distributed. Since the variances appear to be equal and the groups seem to be normally distributed (and we have random/independent samples), we may continue with ANOVA.

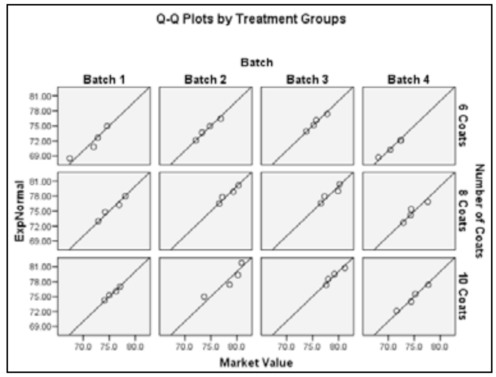
Levene's Test of Equality of Error Variances[a]

Dependent Variable: Market Value

F	df1	df2	Sig.
.4994	11	36	.8909

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

a. Design: Intercept+factor_A+factor_B+factor_A * factor_B



8. You should use the output information in the following manner to answer the question.

Despite thinking that there are no significant interactions (based on profile plot), we must still formally test for significant interaction between the two factors.

Test for significant interaction effects:

Step 1: **Hypotheses**
H_0: There is no interaction between number of coats and batches.
H_a: There is an interaction between number of coats and batches.

Step 2: **Significance Level**
$\alpha = 0.05$

Step 3: **Critical Value(s) and Rejection Region(s)**
Reject the null hypothesis if p-value $\leq$ 0.05.

Step 4: **Construct the ANOVA Table**

Tests of Between-Subjects Effects

Dependent Variable: Market Value

Source	df	Type III Sum of Squares	Mean Square	F	Sig.
factor_A	2	150.3879	75.1940	15.5910	.00001327
factor_B	3	152.8517	50.9506	10.5643	.00003984
factor_A * factor_B	6	1.8521	.3087	.0640	.99882834
Error	36	173.6250	4.8229		
Corrected Total	47	478.7167			

From the output, $F_{AB} = 0.0640$ with 6 and 36 degrees of freedom.
p-value = Sig. = 0.99882834

Step 5: **Conclusion**
Since p-value = 0.99882834 > 0.05 = α, we fail to reject the null hypothesis.

Step 6: **State conclusion in words**
At the $\alpha = 0.05$ significance level, there is not enough evidence to conclude that number of coats and batches have a significant interaction effect on mean value of the imitation pearls.

Since there are no significant interaction effects, we shall test for effects (differences in means) due to the individual factors.

Test for factor A differences:

Step 1: **Hypotheses**

H_0: $\mu_{A1} = \mu_{A2} = \mu_{A3}$

H_a: at least one mean differs from the rest

Step 2: **Significance Level**

$\alpha = 0.05$

Step 3: **Critical Value(s) and Rejection Region(s)**

Reject the null hypothesis if p-value ≤ 0.05.

Step 4: **Construct the ANOVA Table**

Tests of Between-Subjects Effects

Dependent Variable: Market Value

Source	df	Type III Sum of Squares	Mean Square	F	Sig.
factor_A	2	150.3879	75.1940	15.5910	.00001327
factor_B	3	152.8517	50.9506	10.5643	.00003984
factor_A * factor_B	6	1.8521	.3087	.0640	.99882834
Error	36	173.6250	4.8229		
Corrected Total	47	478.7167			

From the output, $F_A = 15.5910$ with 2 and 36 degrees of freedom.

p-value = Sig. = 0.00001327

Step 5: **Conclusion**

Since p-value = 0.00001327 ≤ 0.05 = α, we shall reject the null hypothesis.

Step 6: **State conclusion in words**

At the α = 0.05 significance level, there is enough evidence to conclude that there are differences in mean value of the imitation pearls among the numbers of coats.

Test for factor B differences:

Step 1: **Hypotheses**

H_0: $\mu_{B1} = \mu_{B2} = \mu_{B3} = \mu_{B4}$

H_a: at least one mean differs from the rest

Step 2: **Significance Level**

$\alpha = 0.05$

Step 3: **Critical Value(s) and Rejection Region(s)**

Reject the null hypothesis if p-value ≤ 0.05.

Step 4: **Construct the ANOVA Table**

Tests of Between-Subjects Effects

Dependent Variable: Market Value

Source	df	Type III Sum of Squares	Mean Square	F	Sig.
factor_A	2	150.3879	75.1940	15.5910	.00001327
factor_B	3	152.8517	50.9506	10.5643	.00003984
factor_A * factor_B	6	1.8521	.3087	.0640	.99882834
Error	36	173.6250	4.8229		
Corrected Total	47	478.7167			

From the output, $F_B = 10.5643$ with 3 and 36 degrees of freedom.

p-value = Sig. = 0.00003984

Step 5: **Conclusion**

Since p-value = 0.00003984 ≤ 0.05 = α, we shall reject the null hypothesis.

Step 6: **State conclusion in words**

At the α = 0.05 significance level, there is enough evidence to conclude that there are differences in mean value of the imitation pearls among the batches.

9. Since differences were found in the numbers of coats, we should perform a Tukey-Kramer (Tukey's W) multiple comparison analysis to determine which of the numbers of coats is best. Using the previous output, here is how such an analysis might appear.

Multiple Comparisons

Dependent Variable: Market Value
Tukey HSD

(I) Number of Coats	(J) Number of Coats	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	Upper Bound
6 Coats	8 Coats	-3.6875*	.7764	.0001	-5.5854	-1.7896
	10 Coats	-3.8188*	.7764	.0001	-5.7166	-1.9209
8 Coats	6 Coats	3.6875*	.7764	.0001	1.7896	5.5854
	10 Coats	-.1313	.7764	.9844	-2.0291	1.7666
10 Coats	6 Coats	3.8188*	.7764	.0001	1.9209	5.7166
	8 Coats	.1313	.7764	.9844	-1.7666	2.0291

Based on observed means.
*. The mean difference is significant at the 0.05 level.

Thus, we are 95% confident that 6 coats yields a different (smaller) mean value of the imitation pearls from that when using 8 or 10 coats (these two mean values are similar).

Market Value

Tukey HSD[a,b]

Number of Coats	N	Subset 1	2
6 Coats	16	73.1063	
8 Coats	16		76.7938
10 Coats	16		76.9250
Sig.		1.0000	.9844

Means for groups in homogeneous subsets are displayed.
Based on Type III Sum of Squares
The error term is Mean Square(Error) = 4.823.
a. Uses Harmonic Mean Sample Size = 16.000.
b. Alpha = 0.05.

This table corresponds to our "underline diagram". Note that using 8 or 10 coats (since they are similar) will yield the largest mean value of the pearls.

Since differences were found in the batches, we should perform a Tukey-Kramer (Tukey's W) multiple comparison analysis to determine which of the batches is best. Using the previous output, here is how such an analysis might appear.

Multiple Comparisons

Dependent Variable: Market Value
Tukey HSD

(I) Batch	(J) Batch	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	Upper Bound
Batch 1	Batch 2	-2.7083*	.8966	.0229	-5.1230	-.2937
	Batch 3	-3.4250*	.8966	.0027	-5.8396	-1.0104
	Batch 4	.8333	.8966	.7893	-1.5813	3.2480
Batch 2	Batch 1	2.7083*	.8966	.0229	.2937	5.1230
	Batch 3	-.7167	.8966	.8542	-3.1313	1.6980
	Batch 4	3.5417*	.8966	.0019	1.1270	5.9563
Batch 3	Batch 1	3.4250*	.8966	.0027	1.0104	5.8396
	Batch 2	.7167	.8966	.8542	-1.6980	3.1313
	Batch 4	4.2583*	.8966	.0002	1.8437	6.6730
Batch 4	Batch 1	-.8333	.8966	.7893	-3.2480	1.5813
	Batch 2	-3.5417*	.8966	.0019	-5.9563	-1.1270
	Batch 3	-4.2583*	.8966	.0002	-6.6730	-1.8437

Based on observed means.
*. The mean difference is significant at the 0.05 level.

Thus, we are 95% confident that batches 4 and 1 (similar mean values) yield a different (smaller) mean value of the imitation pearls from that when using batches 2 or 3 (similar mean values).

Market Value

Tukey HSD[a,b]

Batch	N	Subset 1	2
Batch 4	12	73.4500	
Batch 1	12	74.2833	
Batch 2	12		76.9917
Batch 3	12		77.7083
Sig.		.7893	.8542

Means for groups in homogeneous subsets are displayed.
Based on Type III Sum of Squares
The error term is Mean Square(Error) = 4.823.
a. Uses Harmonic Mean Sample Size = 12.000.
b. Alpha = 0.05.

This table corresponds to our "underline diagram". Note that batch method 2 or 3 (since they are similar) will yield the largest mean value of the pearls.